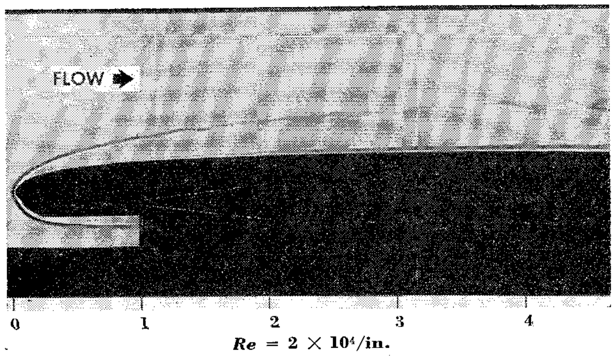

Fig. 4 Instantaneous shadow photograph of the combustion boundary layer (exposure time 5 μsec).

boundary-layer thickness). This distance is not influenced strongly by the Reynolds number. These results are in very good agreement with the behavior predicted by Marxman and Gilbert.[1]

A portion of the schlieren light passing through the test section was blocked off near the downstream end of the slab in Fig. 2, and a photograph of the flame alone was thus obtained. The flame is seen as a luminous streak in the dark right-hand region of Fig. 2. The position of the flame predicted by the schlieren interpretation compared accurately with this direct observation. (Compare Figs. 2 and 3.)

The instantaneous structure of the boundary layer is shown in a 5-μsec exposure shadowgraph that appears in Fig. 4. Transition from a laminar to a turbulent structure occurs at very low Reynolds numbers, apparently even lower than those expected when there is equivalent mass injection without combustion. In Fig. 4 large scale turbulence is visible at the edge of the boundary layer at a Reynolds number of about 2×10^4. Measurements in boundary layers without blowing[3] indicate that transition occurs upstream of the point where this edge turbulence is present. Therefore, transition probably takes place very near the leading edge in Fig. 4 or at a Reynolds number somewhat less than 2×10^4. The effect of combustion may be inferred by noting that Mickley and Davis[4] have reported a transition Reynolds number of 6×10^4 for a noncombustible boundary layer under similar freestream and surface blowing conditions.

The increase in thickening of the boundary layer due to blowing can be observed in Fig. 4 (near the leading edge) by comparing the thickness of the boundary layer on top of the plate, where there is blowing, with that on the bottom, where there is none.

The forementioned experiment simulates a two-dimensional hybrid motor operated at a low-oxidizer flow rate. In a practical range of oxidizer flows for hybrid combustion the transition point occurs even closer to the leading edge, indicating that it is reasonable to approach the problem of hybrid combustion solely from the point of view of a turbulent boundary layer, as stated by Marxman and Gilbert.[1] Further studies are being conducted and will be available for publication at a later date.

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Skin Friction Exerted by a Compressible Fluid Stream on a Flat Plate

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1. Comparison of Existing Theories with Experimental Data

THERE exist about 30 published theories for the calculation of the drag on a flat plate under conditions yielding a turbulent compressible boundary layer. Most of these have been shown to give fair agreement with some of the experimental data. However, it hitherto has been impossible to decide which of the theories gives the best agreement with the complete body of available experimental data.

The authors accordingly have collected as many experimental data as possible (491 values of drag coefficient with associated Reynolds numbers, Mach numbers, and ratios of wall temperature to mainstream temperature) from 22 references. Computer programs have then been written for each of 20 theories; with their aid, the root-mean-square value of the proportional error in drag coefficient has been computed for each theory by reference to all 491 experimental conditions. By proportional error is meant, the experimentally measured

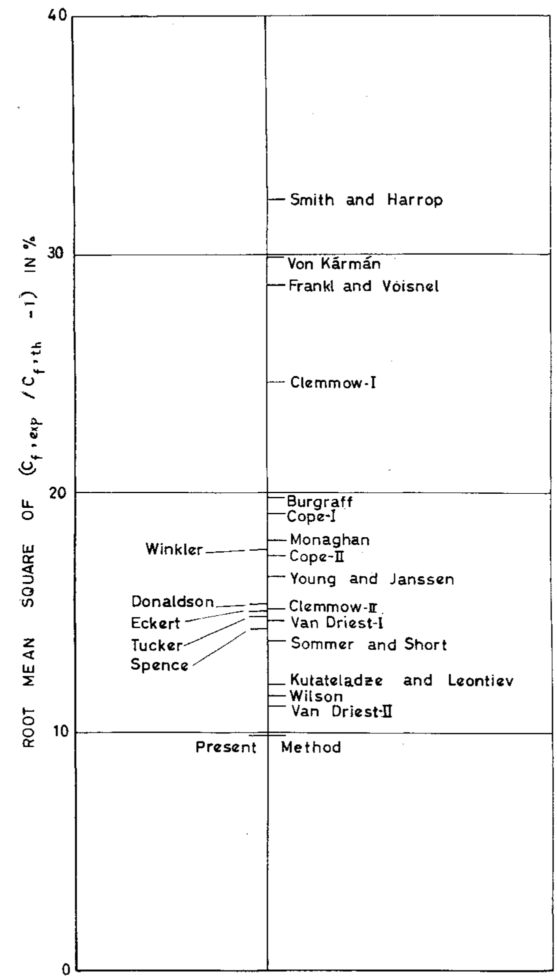

Fig. 1 Comparison of 20 theories with experimental data from 22 sources.

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drag coefficient minus the drag coefficient predicted by the theory in question for the conditions of the experiment, this difference being divided by the latter theoretical drag coefficient.

The "order of merit" of the various theories in respect to accuracy may be deduced from Fig. 1. Of the theories surveyed, the second theory of van Driest[1] proves to give the lowest rms proportional error, namely 11.0%.

The theories that were not included in this study were omitted either because they contained constants for which the authors of the theories had made no recommendation or because use of the theories would have involved an amount of computation which was an order of magnitude greater than that of the most complex theory included in this study, namely that of Kutateladze and Leont'ev.[2] It is of course possible that some of these omitted theories could give even lower rms errors than those represented in Fig. 1.

2. New Calculation Procedure

In an effort to reduce the rms error still further and to make the computation of drag under compressible turbulent conditions an easy one for the designer, a new procedure of calculation has been developed. The essential assumption is that the relations between drag coefficient and Reynolds number which are valid for uniform-property flow continue to hold when properties vary, provided that the drag coefficient is multiplied by a quantity F_c and that the Reynolds number is multiplied by a quantity $F_{R\delta}$ (if the Reynolds number is based on momentum thickness) or F_{Rx}, (if the Reynolds number is based on plate length). The various F's are functions of Mach number and temperature ratio alone.

It has been found that the rms value of the proportional error can be reduced to 9.9% if the following choices are made for the F's:

$$F_c = \left(\frac{T_S}{T_G}\right) \Big/ \left\{\frac{1}{a}\sin^{-1}\frac{2a^2 - b}{(4a^2 + b^2)^{1/2}} + \sin^{-1}\frac{b}{(4a^2 + b^2)^{1/2}}\right\}^2 \quad (1)$$

where

$$a^2 = \left(r\,\frac{\gamma - 1}{2}\,M_G{}^2\right)\Big/\left(\frac{T_S}{T_G}\right)$$

$$b = \left[\left(1 + r\,\frac{\gamma - 1}{2}\,M_G{}^2\right)\Big/\left(\frac{T_S}{T_G}\right)\right] - 1$$

M_G = mainstream Mach number

T_S = wall temperature, °R

T_G = mainstream temperature, °R

γ = specific heat ratio = 1.4 for air

r = recovery factor = 0.89 for Prandtl number $\approx$ 0.7

This function is of course similar to that appearing in the theories of van Driest[1, 3] and many others:

$$F_{R\delta} = T_S/T_G{}^{-0.702}(T_{\mathrm{ad},S}/T_S)^{0.788} \quad (2)$$

where $T_{\mathrm{ad},S}$ is the adiabatic wall temperature (°R). Also,

$$F_{Rx} = F_{R\delta}/F_c \quad (3)$$

The details of the work reported here, together with tables and charts of F_c, $F_{R\delta}$, and other useful functions, will be published elsewhere.[4]

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Contact Surface Tailoring in a Chemical Shock Tube

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This paper presents a method of computing shock tube driver-gas compositions that will give tailoring conditions when the reflected shock wave passes through the contact surface. The method is suitable for any driven gas and any driver-gas mixture for which tailoring is possible. Tailoring data are given for mixtures of hydrogen or helium with argon or nitrogen driving argon. The effects of adding reactant gas to the argon are discussed.

Nomenclature

P = absolute pressure
T = absolute temperature
Cv = constant volume heat capacity per unit mass
E = $Cv \cdot T$, the internal energy per unit mass
γ = specific heat ratio Cp/Cv
α = $(\gamma + 1)/(\gamma - 1)$
β = $(\gamma - 1)/2\gamma$
X = mole fraction of the lower molecular weight component of the driver gas
P_{ij} = P_i/P_j
T_{ij} = T_j/T_i
E_{ij} = E_j/E_i
Subscripts correspond to the gas regions in Fig. 1.

PALMER and Knox[1] have published a useful table of conditions under which contact surface tailoring exists in a chemical shock tube with argon as the shocked gas and mixtures of He-A and N_2-H_2 as driver gases. It has been found that it is not always convenient to use such mixtures especially if the investigation involves analysis of the products of the gaseous reaction for a component present in the driver gas either in bulk or as an impurity. This paper extends the data given by Palmer and Knox to He-N_2 and H_2-A mixtures and indicates the method of solution of the tailoring equations for these and other gas mixtures. The method can be used for driven gases other than argon and for driven gas mixtures.

Computation of the driver-gas composition required to give a particular temperature in the reflected shock region and to give tailored conditions at the contact surface can be resolved into the solution of a number of simultaneous equations. If the specific heat ratios of the components of the driver gas are equal, for example H_2-N_2 or He-A, then solution is simple though tedious. If, however, they are unequal, the equations are implicit and only can be solved by a trial and error method feasible only on a computer.

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